FACULTY OF MATHEMATICAL SCIENCES

DEPARTMENT OF MATHEMATICS

B.SC. (H) MATHEMATICS

Category-I

DISCIPLINE SPECIFIC CORE COURSE – 1: ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

| Course | Credits | Credit d | istribution | of the course | Eligibility | Pre-requisite | |
|---------|---------|----------|-------------|---------------|-------------|---------------|--|
| title & | | Lecture | Tutorial | Practical/ | criteria | of the course | |
| Code | | | | Practice | | (if any) | |
| Algebra | 4 | 3 | 1 | 0 | Class XII | Nil | |
| | | | | | pass with | | |
| | | | | | Mathematics | | |

Learning Objectives

The primary objective of this course is to introduce:

- The basic tools of theory of equations, number theory, and group theory.
- Symmetry group of a plane figure, basic concepts of cyclic groups.
- Classification of subgroups of cyclic groups.

Learning Outcomes:

This course will enable the students to:

- Determine number of positive/negative real roots of a real polynomial.
- Solve cubic and quartic polynomial equations with special condition on roots and in general.
- Employ De-Moivre's theorem in a number of applications to solve numerical problems.
- Use modular arithmetic and basic properties of congruences.
- Recognize the algebraic structure, namely groups, and classify subgroups of cyclic groups.

SYLLABUS OF DSC-1

Theory

Unit – 1

Theory of Equations and Complex Numbers

General properties of polynomials and equations, Fundamental theorem of algebra, Relations between the roots and the coefficients, Upper bounds for the real roots; Theorems on imaginary, integral and rational roots; Newton's method for integral roots, Descartes' rule of

(24 hours)

signs; De-Moivre's theorem for integer and rational indices and their applications, The nth roots of unity, Cardan's solution of the cubic, Descartes' solution of the quartic equation.

Unit - 2

Basic Number Theory

Division algorithm in \mathbb{Z} , Divisibility and the Euclidean algorithm, Fundamental theorem of arithmetic, Modular arithmetic and basic properties of congruences.

Unit – 3

Basics of Group Theory

Groups, Basic properties, Symmetries of a square, Dihedral group, Order of a group, Order of an element, Subgroups, Center of a group, Centralizer of an element, Cyclic groups and properties, Generators of a cyclic group, Classification of subgroups of cyclic groups.

Practical component (if any) - NIL

Essential Readings

- 1. Andreescu, Titu & Andrica, D. (2014). Complex numbers from A to...Z. (2nd ed.). Birkhäuser.
- Dickson, Leonard Eugene (2009). First Course in the Theory of Equations. John Wiley & Sons, Inc. The Project Gutenberg eBook: <u>http://www.gutenberg.org/ebooks/29785</u>
- 3. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.
- 4. Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint 2018.

Suggestive Readings

- Burnside, W.S., & Panton, A.W. (1979), The Theory of Equations, Vol. 1. Eleventh Edition, (Fourth Indian Reprint. S. Chand & Co. New Delhi), Dover Publications, Inc.
- Burton, David M. (2011). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint.
- Rotman, Joseph J. (1995). An Introduction to The Theory of Groups (4th ed.). Springer-Verlag, New York.

(20 hours)

(16 hours)

DISCIPLINE SPECIFIC CORE COURSE – 2: ELEMENTARY REAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

| Course | Credits | Credit | listribution | n of the course | Eligibility | Pre-requisite |
|------------|---------|---------|--------------|-----------------|-------------|---------------|
| title & | | Lecture | Tutorial | Practical/ | criteria | of the course |
| Code | | | | Practice | | (if any) |
| Elementary | | | | | Class XII | NIL |
| Real | 4 | 3 | 1 | 0 | pass with | |
| Analysis | | | | | Mathematics | |

Learning Objectives

The course will develop a deep and rigorous understanding of:

- Real line \mathbb{R} with algebraic.
- Order and completeness properties to prove the results about convergence and divergence of sequences and series of real numbers.

Learning Outcomes

This course will enable the students to:

- Understand the fundamental properties of the real numbers, including completeness and Archimedean, and density property of rational numbers in \mathbb{R} .
- Learn to define sequences in terms of functions from $\mathbb N$ to a subset of $\mathbb R$ and find the limit.
- Recognize bounded, convergent, divergent, Cauchy and monotonic sequences and to calculate the limit superior and limit inferior of a bounded sequence.
- Apply limit comparison, ratio, root, and alternating series tests for convergence and absolute convergence of infinite series of real numbers.

SYLLABUS OF DSC - 2

Theory Unit – 1

(16 hours)

Real Number System

Algebraic and order properties of \mathbb{R} , Absolute value of a real number, Bounded above and bounded below sets, Supremum and infimum of a non-empty subset of \mathbb{R} , The completeness property of \mathbb{R} , Archimedean property, Density of rational numbers in \mathbb{R} .

Unit – 2 Sequences

(24 hours)

Sequences and their limits, Convergent sequence, Limit theorems, Monotone sequences, Monotone convergence theorem, Subsequences, Bolzano-Weierstrass theorem for sequences, Limit superior and limit inferior for bounded sequence, Cauchy sequence, Cauchy's convergence criterion.

Unit – 3 Infinite Series

(20 hours)

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence, Tests for convergence of positive termseries, Integral test, Basic comparison test, Limit comparison test, D'Alembert's ratio test, Cauchy's nth root test, Raabe's test, Alternating series, Leibniz test, Absolute and conditional convergence.

Practical component (if any) - NIL

Essential Readings

- 1. Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
- 2. Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.
- 3. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Suggestive Readings

- Aliprantis C. D., & Burkinshaw, O. (1998). Principles of Real Analysis (3rd ed.). Academic Press.
- Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian reprint.
- Thomson, B. S., Bruckner, A. M., & Bruckner, J. B. (2001). Elementary Real Analysis. Prentice Hall.

DISCIPLINE SPECIFIC CORE COURSE – 3: PROBABILITY AND STATISTICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

| Course | Credits | Credit distribution of the course | | | Eligibility | Pre-requisite of |
|-------------|---------|-----------------------------------|----------|------------|-------------|------------------|
| title & | | Lecture | Tutorial | Practical/ | criteria | the course |
| Code | | | | Practice | | (if any) |
| Probability | | | | | Class XII | NIL |
| and | 4 | 3 | 0 | 1 | pass with | |
| Statistics | | | | | Mathematics | |

Learning Objectives

The Learning Objectives of this course are as follows:

- To make the students familiar with the basic statistical concepts and tools which are needed to study situations involving uncertainty or randomness.
- To render the students to several examples and exercises that blend their everyday experiences with their scientific interests to form the basis of data science.

Learning Outcomes

This course will enable the students to:

- Understand some basic concepts and terminology population, sample, descriptive and inferential statistics including stem-and-leaf plots, dotplots, histograms and boxplots.
- Learn about probability density functions and various univariate distributions such as binomial, hypergeometric, negative binomial, Poisson, normal, exponential and lognormal.
- Understand the remarkable fact that the empirical frequencies of so many natural populati ons, exhibit bell-shaped (i.e., normal) curves, using the Central Limit Theorem.
- Measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.

SYLLABUS OF DSC – 3

Theory

Unit – 1

(15 hours)

Descriptive Statistics, Probability, and Discrete Probability Distributions

Descriptive statistics: Populations, Samples, Stem-and-leaf displays, Dotplots, Histograms, Qualitative data, Measures of location, Measures of variability, Boxplots; Sample spaces and events, Probability axioms and properties, Conditional probability, Bayes' theorem and independent events; Discrete random variables and probability

distributions, Expected values; Probability distributions: Binomial, geometric, hypergeometric, negative binomial, Poisson, and Poisson distribution as a limit.

Unit – 2

Continuous Probability Distributions

Continuous random variables, Probability density functions, Uniform distribution, Cumulative distribution functions and expected values, The normal, exponential and lognormal distributions.

Unit – 3

Central Limit Theorem and Regression Analysis

Sampling distribution and standard error of the sample mean, Central Limit Theorem and applications; Scatterplot of bivariate data, Regression line using principle of least squares, Estimation using the regression lines; Sample correlation coefficient and properties.

Practical (30 hours)

Software labs using Microsoft Excel or any other spreadsheet.

- 1) Presentation and analysis of data (univariate and bivariate) by frequeny tables, descriptive statistics, stem-and-leaf plots, dotplots, histograms, boxplots, comparative boxplots, and probability plots ([1] Section 4.6).
- 2) Fitting of binomial, Poisson and normal distributions.
- 3) Illustrating the Central Limit Theorem through Excel.
- 4) Fitting of regression line using the principle of least squares.
- 5) Computation of sample correlation coefficient.

Essential Reading

1. Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences (9thed.). Cengage Learning India Private Limited. Delhi. Indian Reprint 2020.

Suggestive Reading

• Mood, A. M., Graybill, F. A., & Boes, D. C. (1974). Introduction to the Theory of Statistics (3rd ed.). Tata McGraw-Hill Pub. Co. Ltd. Reprinted 2017.

(15 hours)

DISCIPLINE SPECIFIC CORE COURSE – 4: LINEAR ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

| Cours e title | Credits | Credit o | distributio | n of the course | Eligibilit ycriteria | Pre- requisite of |
|------------------|---------|-------------|--------------|------------------------|-------------------------|----------------------|
| & Code | | Lectur e | Tutoria l | Practical/ Practice | y criteria | the course(if |
| Linear | 4 | 3 | 1 | 0 | Class XII | any) DSC-I: |
| Algebra | | | | | pass with | Algebr |
| | | | | | Mathematic | a |
| | | | | | S | |

Learning Objectives: The objective of the course is to introduce:

- The concept of vectors in \mathbb{R}^n , and their linear independence and dependence.
- Rank and nullity of linear transformations through matrices.
- Various applications of vectors in computer graphics and movements in plane.

Learning Outcomes: This course will enable the students to:

- Visualize the space R^n in terms of vectors and their interrelation with matrices.
- Familiarize with basic concepts in vector spaces, linear independence and span of vectors over a field.
- Learn about the concept of basis and dimension of a vector space.
- Basic concepts of linear transformations, dimension theorem, matrix representation of a linear transformation with application to computer graphics.

SYLLABUS OF DSC-4

UNIT - I: Matrices and System of Linear Equations

Fundamental operations with vectors in Euclidean space R^n , Linear combinations of vectors, Dot product and their properties, Cauchy-Schwarz inequality, Triangle inequality, Solving linear systems using Gaussian elimination, Gauss-Jordan row reduction, Reduced row echelon form, Equivalent systems, Rank and row space, Eigenvalues, Eigenvectors, Eigenspace, Diagonalization, Characteristic polynomial of a matrix, Cayley-Hamilton theorem.

UNIT – II: Introduction to Vector Spaces

Vector spaces, Subspaces, Algebra of subspaces, Linear combination of vectors, Linear span, Linear independence, Bases and dimension, Dimension of subspaces.

UNIT – III: Linear Transformations

Linear transformations, Null space, Range, Rank and nullity of a linear transformation, Matrix representation of a linear transformation, Algebra of linear transformations, Invertibility and

(18 hours)

(15 hours)

(12 hours)

isomorphisms; Application: Computer Graphics-Fundamental movements in a plane, homogenous coordinates, composition of movements.

Essential Readings

- 1. Andrilli, S., & Hecker, D. (2016). *Elementary Linear Algebra* (5th ed.). Elsevier India.
- 2. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2003). *Linear Algebra* (4th ed.). Prentice-Hall of India Pvt. Ltd. New Delhi.

Suggestive Readings

- Lay, David C., Lay, Steven R., & McDonald, Judi J. (2016). *Linear Algebra and itsApplications* (5th ed.). Pearson Education.
- Kolman, Bernard, & Hill, David R. (2001). *Introductory Linear Algebra with Applications* (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.
- Hoffman, Kenneth, & Kunze, Ray Alden (1978). *Linear Algebra* (2nd ed.). Prentice Hallof India Pvt. Limited. Delhi. Pearson Education India Reprint, 2015.

DISCIPLINE SPECIFIC CORE COURSE – 5: CALCULUS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

| Cours | Credits | Credit o | listributio | n of the course | Eligibilit | Pre- |
|----------|---------|----------|-------------|-----------------|-------------|-------------|
| etitle | | Lectur | Tutoria | Practical/ | ycriteria | requisiteof |
| & | | е | 1 | Practice | | the course |
| Code | | | | | | (if any) |
| Calculus | 4 | 3 | 1 | 0 | Class XII | DSC- |
| | | | | | pass | 2: |
| | | | | | with | Elementary |
| | | | | | Mathematics | Real |
| | | | | | | Analysis |

Learning Objectives: The primary objective of this course is:

- To introduce the basic tools of calculus, also known as 'science of variation'.
- To provide a way of viewing and analyzing the real-world.

Learning Outcomes: This course will enable the students to understand:

- The notion of limits, continuity and uniform continuity of functions.
- Geometrical properties of continuous functions on closed and bounded intervals.
- Applications of derivative, relative extrema and mean value theorems.
- Higher order derivatives, Taylor's theorem, indeterminate forms and tracing of curves.

SYLLABUS OF DSC-5

UNIT – I: Limits and Continuity

Limits of functions (ε - δ and sequential approach), Algebra of limits, Squeeze theorem, One-sided limits, Infinite limits and limits at infinity; Continuous functions and its properties on closed and bounded intervals; Uniform continuity.

UNIT – II: Differentiability and Mean Value Theorems

Differentiability of a real-valued function, Algebra of differentiable functions, Chain rule, Relative extrema, Interior extremum theorem, Rolle's theorem, Mean-value theorem and its applications, Intermediate value theorem for derivatives.

UNIT – III:

Successive Differentiation, Taylor's Theorem and Tracing of Plane Curves

Higher order derivatives and calculation of the *n*th derivative, Leibnitz's theorem; Taylor's theorem, Taylor's series expansions of e^x , sin *x*, cos *x*. Indeterminate forms, L'Hôpital's rule; Concavity and inflexion points; Singular points, Asymptotes, Tracing graphs of rational functions and polar equations.

Essential Readings

1. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). John Wiley & Sons Singapore Pvt. Ltd. Reprint (2016) by Wiley India Pvt. Ltd. Delhi.

(15 hours)

(15)

(15 hours)

hours)

- 2. Bartle, Robert G., & Sherbert, Donald R. (2011). *Introduction to Real Analysis* (4th ed.). John Wiley & Sons. Wiley India edition reprint. Prasad, Gorakh (2016). *Differential Calculus* (19th ed.). Pothishala Pvt. Ltd. Allahabad.
- 3. Ross, Kenneth A. (2013). *Elementary Analysis: The Theory of Calculus* (2nd ed.).Undergraduate Texts in Mathematics, Springer. Indian reprint.

Suggestive Readings

- Apostol, T. M. (2007). *Calculus: One-Variable Calculus with an Introduction to Linear Algebra* (2nd ed.). Vol. 1. Wiley India Pvt. Ltd.
- Ghorpade, Sudhir R. & Limaye, B. V. (2006). *A Course in Calculus and Real Analysis*. Undergraduate Texts in Mathematics, Springer (SIE). Indian reprint.

DISCIPLINE SPECIFIC CORE COURSE – 6: ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

| Course | Credits | Credit | listributio | n of the course | Eligibilit | Pre- |
|--------------|---------|--------|-------------|-----------------|------------|-------------|
| title& | | Lectur | Tutoria | Practical/ | ycriteria | requisiteof |
| Code | | e | 1 | Practice | | the course |
| | | | | | | (if any) |
| Ordinary | 4 | 3 | 0 | 1 | Class XII | NI |
| | | | | | | L |
| Differential | | | | | pass with | |
| Equations | | | | | Mathematic | |
| | | | | | S | |

Learning Objectives: The main objective of this course is to introduce the students:

- The exciting world of differential equations.
- Their applications and mathematical modeling.

Learning Outcomes: The course will enable the students to:

- Learn the basics of differential equations and compartmental models.
- Formulate differential equations for various mathematical models.
- Solve first order non-linear differential equations, linear differential equations of higher order and system of linear differential equations using various techniques.
- Apply these techniques to solve and analyze various mathematical models.

SYLLABUS OF DSC-6

UNIT – I: First-Order Differential Equations

Concept of implicit, general and singular solutions for the first order ordinary differential equation; Bernoulli's equation, Exact equations, Integrating factors, Initial value problems, Reducible second order differential equations; Applications of first order differential equations to Newton's law of cooling, exponential growth and decay problems.

UNIT - II: Second and Higher-Order Differential Equations

General solution of homogenous equation of second order, Principle of superposition for a homogenous equation, Wronskian and its properties, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Method of variation of parameters, Method of undetermined coefficients, Two-point boundary value problems, Cauchy- Euler's equation, System of linear differential equations, Application of second orderdifferential equation: Simple pendulum problem.

UNIT - III: Formulation and Analysis of Mathematical Models

Introduction to compartmental models, Lake pollution model; Density-dependent growth model, Interacting population models, Epidemic model of influenza and its analysis, Predator-prey model and its analysis, Equilibrium points, Interpretation of phase plane

Practical (30 hours)- Practical / Lab work to be performed in a Computer Lab:

Modeling of the following problems using ageMath/Mathematica/MATLAB/Maple/Maxima/Scilab etc.

(12 hours)

(15 hours)

(18 hours)

- 1. Solutions of first, second and third order differential equations.
- 2. Plotting of family of solutions of differential equations of first, second and third order.
- 3. Solution of differential equations using method of variation of parameters.
- 4. Growth and decay model (exponential case only).
- 5. Lake pollution model (with constant/seasonal flow and pollution concentration).
- 6. Density-dependent growth model.
- 7. Predatory-prey model (basic Volterra model, with density dependence, effect ofDDT, two prey one predator).
- 8. Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).

Essential Readings

- 1. Barnes, Belinda & Fulford, Glenn R. (2015). *Mathematical Modeling with Case Studies*, Using Maple and MATLAB (3rd ed.). CRC Press. Taylor & Francis Group.
- 2. Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). *Differential Equations and Boundary Value Problems: Computing and Modeling* (5th ed.). Pearson Education.
- 3. Ross, Shepley L. (2014). *Differential Equations* (3rd ed.). Wiley India Pvt. Ltd.

Suggestive Reading

• Simmons, George F. (2017). *Differential Equations with Applications and HistoricalNotes* (3rd ed.). CRC Press. Taylor & Francis Group.

DISCIPLINE SPECIFIC CORE COURSE -7: GROUP THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

| Course | Credits | Credit d | listribution | of the course | Eligibility | Pre-requisite | |
|-----------------|---------|----------|--------------|------------------------|---------------------------------------|---------------------------|--|
| title & Code | | Lecture | Tutorial | Practical/ Practice | criteria | of the course (if any) | |
| Group Theory | 4 | 3 | 1 | 0 | Class XII pass with Mathematics | Algebra | |

Learning Objectives

The primary objective of this course is to introduce:

- Symmetric groups, normal subgroups, factor groups, and direct products of groups.
- The notions of group homomorphism to study the isomorphism theorems with applications.
- Classification of groups with small order according to isomorphisms.

Learning Outcomes

This course will enable the students to:

- Analyse the structure of 'small' finite groups, and examine examples arising as groups of permutations of a set, symmetries of regular polygons.
- Understand the significance of the notion of cosets, Lagrange's theorem and its consequences.
- Know about group homomorphisms and isomorphisms and to relate groups using these mappings.
- Express a finite abelian group as the direct product of cyclic groups of prime power orders.
- Learn about external direct products and its applications to data security and electric circuits.

SYLLABUS OF DSC - 7

Unit – 1

Permutation Groups, Lagrange's Theorem and Normal Subgroups

Permutation groups and group of symmetries, Cycle notation for permutations and properties, Even and odd permutations, Alternating groups; Cosets and its properties, Lagrange's theorem and consequences including Fermat's Little theorem, Number of elements in product of two finite subgroups; Normal subgroups, Factor groups, Cauchy's theorem for finite Abelian groups.

Unit – 2

Group Homomorphisms and Automorphisms

Group homomorphisms, isomorphisms and properties, Cayley's theorem; First, Second and Third isomorphism theorems for groups; Automorphism, Inner automorphism, Automorphism

(18 hours)

groups, Automorphism groups of cyclic groups, Applications of factor groups to automorphism groups.

(12 hours)

Unit – 3

Direct Products of Groups and Fundamental Theorem of Finite Abelian Groups

External direct products of groups and its properties, The group of units modulo n as an external direct product, Applications to data security and electric circuits; Internal direct products; Fundamental theorem of finite abelian groups and its isomorphism classes.

Essential Reading

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.

Suggestive Readings

- Artin, Michael. (1991). Algebra (2nd ed.). Pearson Education. Indian Reprint 2015.
- Dummit, David S., & Foote, Richard M. (2016). Abstract Algebra (3rd ed.). Student Edition. Wiley India.
- Herstein, I. N. (1975). Topics in Algebra (2nd ed.). Wiley India, Reprint 2022.
- Rotman, Joseph J. (1995). An Introduction to The Theory of Groups (4th ed.). Springer-Verlag, New York.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC CORE COURSE -8: RIEMANN INTEGRATION

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

| Course | Credits | Credit dis | stribution o | of the course | Eligibility | Pre-requisite of the course (if any) |
|------------------------|---------|------------|--------------|------------------------|---------------------------------------|--|
| title & Code | | Lecture | Tutorial | Practical/ Practice | criteria | |
| Riemann Integration | 4 | 3 | 1 | 0 | Class XII pass with Mathematics | Elementary Real Analysis, and Calculus |

Learning Objectives

The primary objective of this course is to:

- Understand the integration of bounded functions on a closed and bounded interval and its extension to the cases where either the interval of integration is infinite, or the integrand has infinite limits at a finite number of points on the interval of integration.
- Learn some of the properties of Riemann integrable functions, its generalization and the applications of the fundamental theorems of integration.
- Get an exposure to the utility of integration for practical purposes.

Learning Outcomes

This course will enable the students to:

- Learn about some of the classes and properties of Riemann integrable functions, and the applications of the Riemann sums to the volume and surface of a solid of revolution.
- Get insight of integration by substitution and integration by parts.
- Know about convergence of improper integrals including, beta and gamma functions.

SYLLABUS OF DSC - 8

Unit – 1

The Riemann Integral

Definition of upper and lower Darboux sums, Darboux integral, Inequalities for upper and lower Darboux sums, Necessary and sufficient conditions for the Darboux integrability; Riemann's definition of integrability by Riemann sum and the equivalence of Riemann's and Darboux's definitions of integrability; Definition and examples of the Riemann-Stieltjes integral.

Unit – 2

Properties of The Riemann Integral and Fundamental Theorems

Riemann integrability of monotone functions and continuous functions, Properties of Riemann integrable functions; Definitions of piecewise continuous and piecewise monotone functions and their Riemann integrability; Intermediate value theorem for integrals, Fundamental Theorems of Calculus (I and II).

Unit – 3

Applications of Integrals and Improper Integrals

Methods of integration: integration by substitution and integration by parts; Volume by slicing and cylindrical shells, Length of a curve in the plane and the area of surfaces of revolution. Improper integrals of Type-I, Type-II and mixed type, Convergence of improper integrals, The beta and gamma functions and their properties.

Essential Readings

- 1. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer.
- 2. Anton, Howard, Bivens Irl and Davis Stephens (2012). Calculus (10th edn.). John Wiley & Sons, Inc.
- 3. Denlinger, Charles G. (2011). Elements of Real Analysis, Jones & Bartlett India Pvt. Ltd., Indian Reprint.
- 4. Ghorpade, Sudhir R. and Limaye, B. V. (2006). A Course in Calculus and Real Analysis. Undergraduate Texts in Mathematics, Springer (SIE). Indian Reprint.

Suggestive Readings

- Bartle, Robert G., & Sherbert, Donald R. (2015). Introduction to Real Analysis (4th ed.). Wiley, Indian Edition.
- Kumar Ajit and Kumaresan S. (2014). A Basic Course in Real Analysis. CRC Press, Taylor & Francis Group, Special Indian Edition.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

(18 hours)

(12 hours)

DISCIPLINE SPECIFIC CORE COURSE- 9: DISCRETE MATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

| Course title | | | Eligibility | Pre-requisite | | |
|-------------------------|---|---------|-------------|------------------------|---------------------------------------|-------------------------------|
| & Code | | Lecture | Tutorial | Practical/ Practice | criteria | of the course (if any) |
| Discrete Mathematics | 4 | 3 | 0 | 1 | Class XII pass with Mathematics | Algebra and Linear Algebra |

Learning Objectives

The primary objective of the course is to:

- Make students embark upon a journey of enlightenment, starting from the abstract concepts in mathematics to practical applications of those concepts in real life.
- Make the students familiar with the notion of partially ordered set and a level up with the study of lattice, Boolean algebra and related concepts.
- Culminate the journey of learning with practical applications using the knowledge attained from the abstract concepts learnt in the course.

Learning Outcomes

This course will enable the students to:

- Understand the notion of partially ordered set, lattice, Boolean algebra with applications.
- Handle the practical aspect of minimization of switching circuits to a great extent with the methods discussed in this course.
- Apply the knowledge of Boolean algebras to logic, set theory and probability theory.

SYLLABUS OF DSC - 9

Unit – 1

Cardinality and Partially Ordered Sets

The cardinality of a set; Definitions, examples and basic properties of partially ordered sets, Order-isomorphisms, Covering relations, Hasse diagrams, Dual of an ordered set, Duality principle, Bottom and top elements, Maximal and minimal elements, Zorn's lemma, Building new ordered sets, Maps between ordered sets.

Unit – 2

Lattices

Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products, Lattice isomorphism; Definitions, examples and properties of modular and distributive lattices; The $M_3 - N_5$ theorem with applications, Complemented lattice, Relatively complemented lattice, Sectionally complemented lattice.

Unit – 3

Boolean Algebras and Applications

Boolean algebras, De Morgan's laws, Boolean homomorphism, Representation theorem, Boolean polynomials, Boolean polynomial functions, Equivalence of Boolean polynomials, Disjunctive normal form and conjunctive normal form of Boolean polynomials; Minimal forms

(15 hours)

(15 hours)

of Boolean polynomials, Quine-McCluskey method, Karnaugh diagrams, Switching circuits and applications, Applications of Boolean algebras to logic, set theory and probability theory.

Practical (30 hours):

Practical/Lab work to be performed in a computer Lab using any of the Computer Algebra System Software such as Mathematica/MATLAB /Maple/Maxima/Scilab/SageMath etc., for the following problems based on:

- 1) Expressing relations as ordered pairs and creating relations.
- 2) Finding whether or not, a given relation is:
 - i. Reflexive ii. Antisymmetric iii. Transitive iv. Partial order
- 3) Finding the following for a given partially ordered set
 - i. Covering relations.
 - ii. The corresponding Hasse diagram representation.
 - iii. Minimal and maximal elements.
- 4) Finding the following for a subset S of a given partially ordered set P
 - i. Whether a given element in *P* is an upper bound (lower bound) of *S* or not.
 - ii. Set of all upper bounds (lower bounds) of *S*.
 - iii. The least upper bound (greatest lower bound) of *S*, if it exists.
- 5) Creating lattices and determining whether or not, a given partially ordered set is a lattice.
- 6) Finding the following for a given Boolean polynomial function:
 - i. Representation of Boolean polynomial function and finding its value when the Boolean variables in it take particular values over the Boolean algebra {0,1}.
 - ii. Display in table form of all possible values of Boolean polynomial function over the Boolean algebra {0,1}.
- 7) Finding the following:
 - i. Dual of a given Boolean polynomial/expression.
 - ii. Whether or not two given Boolean polynomials are equivalent.
 - iii. Disjunctive normal form (Conjunctive normal form) from a given Boolean expression.
 - iv. Disjunctive normal form (Conjunctive normal form) when the given Boolean polynomial function is expressed by a table of values.
- 8) Representing a given circuit diagram (expressed using gates) in the form of Boolean expression.
- 9) Minimizing a given Boolean expression to find minimal expressions.

Essential Readings

- 1. Davey, B. A., & Priestley, H. A. (2002). Introduction to Lattices and Order (2nd ed.). Cambridge University press, Cambridge.
- 2. Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.
- 3. Lidl, Rudolf & Pilz, Gunter. (2004). Applied Abstract Algebra (2nd ed.), Undergraduate Texts in Mathematics. Springer (SIE). Indian Reprint.

Suggested Readings

- Donnellan, Thomas. (1999). Lattice Theory (1st ed.). Khosla Pub. House. Indian Reprint.
- Rosen, Kenneth H. (2019). Discrete Mathematics and its Applications (8th ed.), Indian adaptation by Kamala Krithivasan. McGraw-Hill Education. Indian Reprint 2021.

B.Sc. (Hons) Mathematics, Semester-III, DSE-Courses

DISCIPLINE SPECIFIC ELECTIVE COURSE -1(i): GRAPH THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

| Course | Credits | Credit d | istribution | Eligibility | Pre-requisite | |
|-----------------|---------|----------|-------------|------------------------|---------------------------------------|---------------------------|
| title & Code | | Lecture | Tutorial | Practical/ Practice | criteria | of the course (if any) |
| Graph Theory | 4 | 3 | 1 | 0 | Class XII pass with Mathematics | Nil |

Learning Objectives

The primary objective of this course is to introduce:

- Problem-solving techniques using various concepts of graph theory.
- Various properties like planarity and chromaticity of graphs.
- Several applications of these concepts in solving practical problems.

Learning Outcomes

This course will enable the students to:

- Learn modelling of real-world problems by graphs.
- Know characteristics of different classes of graphs.
- Learn representation of graphs in terms of matrices.
- Learn algorithms to optimize a solution.
- Understand some properties of graphs and their applications in different practical situations.

SYLLABUS OF DSE - 1(i)

Unit – 1

Graphs, Paths and Circuits

Definition, Examples and basic properties of graphs, Subgraphs, Pseudographs, Complete graphs, Bipartite graphs, Isomorphism of graphs, Paths and circuits, Connected graphs, Eulerian circuits, Hamiltonian cycles, Adjacency matrix, Weighted graph, Travelling salesman problem, Shortest path, Dijkstra's algorithm.

Unit – 2

Applications of Paths and Circuits, Trees

Applications of Path and Circuits: The Chinese Postman Problem, Digraphs, Bellman-Ford Algorithm, Tournaments, Scheduling Problem, Trees, Properties of Trees, Spanning Trees, Minimum Spanning Tree Algorithms.

Unit – **3**

Connectivity and Graph Coloring, Planar Graphs

Cut-vertices, Blocks and their Characterization, Connectivity and edge-connectivity, Planar graphs, Euler's formula, Kuratowski theorem, Graph coloring and applications, Matchings, Hall's theorem, Independent sets and covers.

(12 hours)

(18 hours)

Essential Readings

- 1. Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.
- 2. Chartrand, Gary, & Zhang, Ping (2012). A First Course in Graph Theory. Dover Publications.

Suggestive Readings

- Bondy, J. A., and Murty, U.S.R. (2008). Graph Theory. Graduate Texts in Mathematics, Springer.
- Diestel, Reinhard (2017). Graph Theory (5th ed.). Graduate Texts in Mathematics, Springer.
- West, Douglas B. (2001). Introduction to Graph Theory (2nd ed.). Prentice Hall. Indian Reprint.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC ELECTIVE COURSE- 1(ii): MATHEMATICAL PYTHON

| Course title | Credits | Credit d | istribution | of the course | Eligibility | Pre-requisite |
|------------------------|---------|----------|-------------|------------------------|---------------------------------------|---------------------------------|
| & Code | | Lecture | Tutorial | Practical/ Practice | criteria | of the course (if any) |
| Mathematical Python | 4 | 3 | 0 | 1 | Class XII pass with Mathematics | Basic knowledge of Python |

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Learning Objectives

The Learning Objectives of this course are as follows:

- To be able to model and solve mathematical problems using Python Programs.
- To experience utility of open-source resources for numerical and symbolic mathematical software systems.

Learning Outcomes

This course will enable the students to use Python:

- For numerical and symbolic computation in mathematical problems from calculus, algebra, and geometry.
- To tabulate and plot diverse graphs of functions and understand tracing of shapes, geometries, and fractals.
- To prepare smart documents with LaTeX interface.

SYLLABUS OF DSE - 1(ii)

Theory

Unit – 1

Drawing Shapes, Graphing and Visualization

Drawing diverse shapes using code and Turtle; Using matplotlib and NumPy for data organization, Structuring and plotting lines, bars, markers, contours and fields, managing subplots and axes; Pyplot and subplots, Animations of decay, Bayes update, Random walk.

Unit – 2

Numerical and Symbolic Solutions of Mathematical Problems

NumPy for scalars and linear algebra on *n*-dimensional arrays; Computing eigenspace, Solving dynamical systems on coupled ordinary differential equations, Functional programming fundamentals using NumPy; Symbolic computation and SymPy: Differentiation and integration of functions, Limits, Solution of ordinary differential equations, Computation of eigenvalues, Solution of expressions at multiple points (lambdify), Simplification of expressions, Factorization, Collecting and canceling terms, Partial fraction decomposition, Trigonometric simplification, Exponential and logarithms, Series expansion and finite differences, Solvers, Recursive equations.

Unit – 3

Document Generation with Python and LaTeX

Pretty printing using SymPy; Pandas API for IO tools: interfacing Python with text/csv, HTML, LaTeX, XML, MSExcel, OpenDocument, and other such formats; Pylatex and writing document files from Python with auto-computed values, Plots and visualizations.

Practical (30 hours):Software labs using IDE such as Spyder and Python Libraries.

- Installation, update, and maintenance of code, troubleshooting.
- Implementation of all methods learned in theory.
- Explore and explain API level integration and working of two problems with standard Python code.

Essential Readings

- 1. Farrell, Peter (2019). Math Adventures with Python. No Starch Press. ISBN Number: 978-1-59327-867-0.
- 2. Farrell, Peter and et al. (2020). The Statistics and Calculus with Python Workshop. Packet Publishing Ltd. ISBN: 978-1-80020-976-3.
- 3. Saha, Amit (2015). Doing Math with Python. No Starch Press. ISBN: 978-1-59327-640-9 **Suggested Readings**
- Morley, Sam (2022). Applying Math with Python (2nd ed.). Packet Publishing Ltd. ISBN: 978-1-80461-837-0
- Online resources and documentation on the libraries, such as:
 - o https://matplotlib.org
 - o https://sympy.org
 - $\circ \quad https://pandas.pydata.org$
 - \circ https://numpy.org
 - https://pypi.org
 - o https://patrickwalls.github.io/mathematicalpython/

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

(15 hours)

(18 hours)

(12 hours)

DISCIPLINE SPECIFIC ELECTIVE COURSE-1(iii): NUMBER THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

| Course | Credits | Credit d | istribution | of the course | Eligibility criteria | Pre-requisite of the course (if any) |
|------------------|---------|----------|-------------|------------------------|---------------------------------------|--|
| title & Code | | Lecture | Tutorial | Practical/ Practice | | |
| Number Theory | 4 | 3 | 1 | 0 | Class XII pass with Mathematics | Algebra |

Learning Objectives

The primary objective of this course is to introduce:

- The number theoretic techniques of computations with the flavour of abstraction.
- The Euclidean algorithm, linear Diophantine equations, congruence equations, arithmetic functions and their applications, Fermat's little, Euler's and Wilson's theorems.
- Primitive roots, quadratic residues and nonresidues, the Legendre symbol and the law of Quadratic Reciprocity.
- Introduction to cryptography, public-key cryptosystems and applications.

Learning Outcomes

This course will enable the students to:

- Use modular arithmetic in solving linear and system of linear congruence equations.
- Work with the number theoretic functions, their properties and their use.
- Learn the forms of positive integers that possess primitive roots and the Quadratic Reciprocity Law which deals with the solvability of quadratic congruences.
- Understand the public-key cryptosystems, in particular, RSA.

SYLLABUS OF DSE - 1(iii)

Unit – 1

Linear Diophantine equation and Theory of Congruences

The Euclidean Algorithm and linear Diophantine equation; Least non-negative residues and complete set of residues modulo n; Linear congruences, The Chinese remainder theorem and system of linear congruences in two variables; Fermat's little theorem, Wilson's theorem and its converse, Application to solve quadratic congruence equation modulo odd prime p.

Unit – 2

Number-Theoretic Functions and Primitive Roots

Number-theoretic functions for the sum and number of divisors, Multiplicative function, Möbius inversion formula and its properties; Greatest integer function with an application to the calendar; Euler's Phi-function, Euler's theorem and some properties of the Phi-function; The order of an integer modulo n and primitive roots for primes, Primitive roots of composite numbers n: when n is of the form 2^k , and when n is a product of two coprime numbers.

(12 hours)

(21 hours)

Unit – 3

(12 hours)

Quadratic Reciprocity Law and Public Key Cryptosystems

The quadratic residue and nonresidue of an odd prime and Euler's criterion, The Legendre symbol and its properties, Quadratic Reciprocity law and its application; Introduction to cryptography, Hill's cipher, Public-key cryptography and RSA.

Essential Reading

1. Burton, David M. (2011). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint 2017.

Suggestive Readings

- Andrews, George E. (1994). Number Theory. Dover publications, Inc. New York.
- Robbins, Neville (2007). Beginning Number Theory (2nd ed.). Narosa Publishing House Pvt. Ltd. Delhi.
- Rosen, Kenneth H. (2011). Elementary Number Theory and its Applications (6th ed.). Pearson Education. Indian Reprint 2015.